THEORETICAL INVESTIGATION OF NONISOTHERMAL FLOW OF A RAREFIED GAS IN A CYLINDRICAL CAPILLARY

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This paper calculates thermocreep and Poiseuille flow in a cylindrical capillary using the Bubnov–Galerkin method. Asymptotic formulas are obtained for flow in the viscous regime with slip.

In a theoretical investigation of isothermal and nonisothermal flow of a rarefied gas in a cylindrical capillary in [1, 2], the kinetic equation was used with the BGK intermolecular collision operator model. In both cases the operator model was used with unit collision frequency, determining the gas rarefaction parameter δ only in terms of the viscosity. However, as was noted in [3], the use of unit collision frequency does not enable one to correctly describe cross effects, due simultaneously to the gas viscosity and thermal conductivity. In addition, as is shown by calculations, in this case the thermal slip velocity of the gas near the wall is not the same as the value obtained from direct solution of the kinetic equation with the Boltzmann operator for intermolecular collisions [4].

Since references [1, 2, 5] have dealt in sufficient detail with the statement of the problem and the procedure for obtaining integral transfer equations for the functions $\psi_p(r)$ and $\psi_T(r)$, which determine the macroscopic velocities $U_{p,T}(r)$ and the dimensionless Poiseuille flow $Q_p(\delta_p)$ and thermocreep $Q_T(\delta_T)$, we restrict ourselves here to writing these equations in the form

$$\psi_k(r) = \Phi_k(r) + \frac{\delta_k}{\pi} \iint_{\Sigma_1} \psi_k(\mathbf{r}') K_0(\mathbf{r}, \mathbf{r}') \frac{d\mathbf{r}'}{|\mathbf{r}' - \mathbf{r}|} + \frac{\delta_k}{\pi} \iint_{\Sigma_2} \psi_k(\mathbf{r}') K_0(\mathbf{r}, \mathbf{r}', \varepsilon_k) \frac{d\mathbf{r}'}{|\mathbf{r}' - \mathbf{r}_N|}, \quad k = p, \text{T}. \tag{1}$$

Here the free terms $\Phi_k(\mathbf{r})$ have the form

$$\Phi_p(r) = 1, \quad \Phi_\text{T}(r) = 1 - \frac{\delta_\text{T}}{\pi} \iint_{\Sigma_1} K_2(\mathbf{r}, \mathbf{r}') \frac{d\mathbf{r}'}{|\mathbf{r}' - \mathbf{r}|} - \frac{\delta_\text{T}}{\pi} \iint_{\Sigma_2} K_2(\mathbf{r}, \mathbf{r}', \varepsilon_\text{T}) \frac{d\mathbf{r}'}{|\mathbf{r}' - \mathbf{r}_N|}. \tag{2}$$

The rarefaction parameters δ_p and δ_T in Eq. (2) are defined as $\delta_{p,T} = (\sqrt{\pi}/2)(R/\lambda_{p,T})$, where λ_p and λ_T are the average molecular mean free paths, calculated from the viscosity and the translational part of the thermal conductivity, respectively [4]. The parameters δ_p and δ_T are linked by the relation $\delta_T = 2\delta_p/3$.

The integration areas Σ_1 and Σ_2 in Eqs. (1) and (2) are determined by the intervals of integration over the polar angle α in the limits $0 \leq \alpha \leq 2$, and in the variable s in the ranges $0 \leq s \leq s_0$ and $0 \leq s \leq l$, respectively, where s_0 and l are the lengths of the intercept SM and the chord MN (see Fig. 1 in [1, 5]). A family of kernels for the integral equations has been introduced into Eqs. (1) and (2) of the type

$$K_n(\mathbf{r}, \mathbf{r}') = \int_0^\infty c^n \exp\left(-c^2 - \frac{\delta_k |\mathbf{r}' - \mathbf{r}|}{c}\right) dc,$$

$$K_n(\mathbf{r}, \mathbf{r}', \varepsilon_k) = \int_0^\infty \frac{(1 - \varepsilon_k) \exp(-\delta_k |\mathbf{r}_M - \mathbf{r}|/c)}{1 - (1 - \varepsilon_k) \exp(-\delta_k |\mathbf{r}_M - \mathbf{r}_N/c)} c^n \exp\left(-c^2 - \frac{\delta_k |\mathbf{r}' - \mathbf{r}_N|}{c}\right) dc, \quad k = p, \text{T}. \tag{3}$$

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It should be noted that an incorrect statement has been assumed in [1, 5] to simplify writing, in setting up the integral equation of transfer type, Eq. (1). However, all the computations were based on the solution of the equations in the form of Eq. (1).

Two accommodation coefficients for the tangential momentum ε_p and ε_T have been used in Eqs. (1)-(3). The need to consider two accommodation coefficients arises from the fact that in the nonisothermal gas flow, in contrast with the isothermal flow, the temperature of molecules reflected from the wall differs from the wall temperature, because of a tangential temperature gradient (and, possibly, a radial gradient). In this case one must introduce the nonisothermal coefficient ε_T, from the definition of the Maxwellian kernel for molecular scattering at the wall [6] and the accommodation coefficient for tangential momentum [3].

The dimensionless Poiseuille flux $Q_p(\delta_p)$ and the thermocreep flux $Q_T(\delta_T)$ are determined, as in [1, 2, 5], from the formulas

$$Q_k(\delta_k) = -\frac{1}{\delta_k}\left[1 - 2\int_0^1 \psi_k(r)\,r\,dr\right], \quad k = p,\ \mathrm{T}. \tag{4}$$

To solve the integral equations (1) it is convenient to go to a polar coordinate system with variables α and s and to introduce the family of integral operators of the type

$$\hat{K}_n\psi_k = \int_0^{2\pi} \hat{B}_n\psi_k\,d\alpha, \tag{5}$$

$$\hat{B}_n\psi_k = \int_0^{s_0} \psi_k(s)\,ds \int_0^{\infty} c^n \exp\left(-c^2 - \frac{\delta_k s}{c}\right) dc + \int_0^{l} \psi_k(s)\,ds \int_0^{\infty} c^n \frac{(1-\varepsilon_k)\exp[-c^2 - \delta_k(s+s_0)/c]}{1-(1-\varepsilon_k)\exp(-\delta_k l/c)}\,dc, \quad k = p,\ \mathrm{T}. \tag{6}$$

Then we can write Eq. (1) as

$$\psi_p(r) = 1 + \frac{\delta_p}{\pi}\hat{K}_0\psi_p, \quad \psi_\mathrm{T}(r) = 1 - \frac{\delta_\mathrm{T}}{\pi}\hat{K}_2 1 + \frac{\delta_\mathrm{T}}{\pi}\hat{K}_2\psi_\mathrm{T}. \tag{7}$$

Integral equations (7) belong to the class of Fredholm equations of the second kind. Therefore, they can be solved using one of the approximate methods of solution – the Bubnov–Galerkin method [7].

We shall choose a system of base functions $\varphi_i(r)$ ($i = 0, 1, 2, \ldots$). From the symmetry of the problem considered, it follows that the base functions $\varphi_i(r)$ must be even functions $r = |\mathbf{r}|$. Then the desired functions can be represented in the form of infinite series with the corresponding expansion coefficients

$$\psi_k(r) = \sum_{i=0}^{\infty} C_{ki}\varphi_i(r) = \sum_{i=0}^{\infty} C_{ki} r^{2i}, \quad k = p,\ \mathrm{T}. \tag{8}$$

Results were obtained in [1, 2, 5] of an investigation using the second approximation of the Bubnov–Galerkin ($i = 0, 1$) method. The rate of convergence of the method in the different approximations remains unknown. Therefore, the solution is described briefly here in the third approximation, i.e., we seek the functions $\psi^{(3)}{}_k$ in the form

$$\psi_k^{(3)}(r) = A_k + B_k r^2 + C_k r^4. \tag{9}$$

If we substitute Eq. (9) into Eq. (7), it gives a discrepancy which is typical of the difference between the exact solution of Eqs. (7) and the approximate solution, of the following form:

$$R_k(r, A_k, B_k, C_k) = \psi_k^{(3)}(r) - \Phi_k(r) - \frac{\delta_k}{\pi}\hat{K}_n\psi_k^{(3)}(r), \quad k = p,\ \mathrm{T}. \tag{10}$$

To obtain an approximate solution, we need to choose coefficients A_k, B_k, and C_k in such a way that the discrepancy R_k should be a minimum. One can obtain the least discrepancy only when the discrepancies are required to be orthogonal to the base functions (1, r^2, r^4), i.e.,

$$(R_k, 1) = 0, \quad (R_k, r^2) = 0, \quad (R_k, r^4) = 0. \tag{11}$$

Here and below the scalar product (f, g) in the Hilbert subspace F of the space $L_2(D)$, where the region of integration is the interval [0, 1], is given by the formula

$$(f, g) = 2\pi \int_0^1 f(r)\, g(r)\, r dr. \tag{12}$$

Thus, from Eq. (11), taking account of Eq. (10), we obtain the following system of nonuniform algebraic equations to determine the expansion coefficients A_k, B_k, and C_k:

$$\begin{Vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{Vmatrix} \cdot \begin{Vmatrix} A_k \\ B_k \\ C_k \end{Vmatrix} = \begin{Vmatrix} d_{1k} \\ d_{2k} \\ d_{3k} \end{Vmatrix}, \quad k = p, \text{T}, \tag{13}$$

where

$$\begin{gathered} d_{1p} = \pi, \quad d_{2p} = \frac{\pi}{2}, \quad d_{3p} = \frac{\pi}{3}, \quad d_{1\text{T}} = \pi - \frac{\delta_\text{T}}{\pi}(1, \hat{K}_2 1), \\ d_{2\text{T}} = \frac{\pi}{2} - \frac{\delta_\text{T}}{\pi}(r^2, \hat{K}_2 1), \quad d_{3\text{T}} = \frac{\pi}{3} - \frac{\delta_\text{T}}{\pi}(r^4, \hat{K}_2 1). \end{gathered} \tag{14}$$

The coefficients a_{ij} of the system matrix, Eq. (13), can easily be obtained in the form of Eq. (14), when one uses the definition of the scalar product of the two functions, Eq. (12). From the symmetry of kernels of the integral operators it follows that the coefficients of the matrix a_{ij} are symmetric, i.e.,

$$a_{21} = a_{12}, \quad a_{32} = a_{23}, \quad a_{31} = a_{13}. \tag{15}$$

The procedure in calculating the coefficients a_{ij} and d_{ik} is extremely complex and cumbersome. Therefore, we shall describe only the main outline. Since the inequality $(1 - \varepsilon_k) \exp(-\delta_k l/c) \leq 1$ holds for any values of the rarefaction parameter δ_k and the accommodation coefficient ε_k, then the integral operator $\hat{B}_n\psi_k$ of Eq. (6) can be written in the form of the series

$$\hat{B}_n\psi_k = \int_0^{s_0} \psi_k(s)\, ds \int_0^\infty c^n \exp(-c^2 - \delta_k s/c)\, dc + \sum_{i=1}^\infty (1 - \varepsilon_k)^i \int_0^l \psi_k(s)\, ds \int_0^\infty c^n \exp\left\{-c^2 - \frac{\delta_k}{c}\,[(i-1)\,l + s + s_0]\right\} dc. \tag{16}$$

Using Eq. (16) we can write the coefficients a_{ij} and d_{ik} in the form of infinite series.

The coefficients a_{ij} were calculated on the M-222 computer. To reduce the machine time required to calculate the double integrals appearing in a_{ij}, a procedure was used for calculating integrals of Abramovich type [8]. This procedure, used to expand the subintegral functions in a series in small values of the parameter $t \leq 2.5$, and the asymptotic representation of the integrals for large values of t [8], gives an accuracy in calculating the integrals of no worse than 0.1%. In addition, recurrence relations are used for the integrals in calculating a_{ij} and d_{ik}, to avoid a nonintegrable singularity in the subintegral function containing $(1 - t^2)^{-1/2}$ when $t \to 1$.

The results of the calculations of the dimensionless Poiseuille flux $Q_p(\delta_p)$ and the thermocreep flux $Q_T(\delta_T)$ are shown in Tables 1 and 2 for various values of the rarefaction parameters ε_p and ε_T. All the results were determined with an accuracy of no worse than 0.1%.

Comparison of the results presented with results of the second approximation of the Bubnov–Galerkin method indicate that the convergence is rapid. The difference in the results for Q_p and Q_T in the second and third approximations does not exceed 1%. However, it should be noted that, while the data for the two approximations practically coincide for small $\delta_{p,T}$, the difference increases monotonically for large $\delta_{p,T} > 1$, especially in the viscous flow region. This divergence can be explained, evidently, by the fact that it is dif-

TABLE 1. Values of the Dimensionless Poiseuille Flux for Various Values of the Rarefaction Parameter δ_p and the Isothermal Accommodation Coefficient for Tangential Momentum ε_p

δ_p	ε_p			
	1	0,9	0,8	0,6
0,01	1,476	1,791	2,187	3,374
0,02	1,460	1,764	2,144	3,255
0,04	1,438	1,728	2,085	3,137
0,06	1,423	1,703	2,045	3,044
0,08	1,412	1,686	2,015	2,992
0,1	1,403	1,669	1,993	2,944
0,2	1,381	1,628	1,933	2,789
0,4	1,379	1,617	1,890	2,720
0,6	1,398	1,623	1,890	2,691
0,8	1,425	1,640	1,906	2,693
1	1,457	1,668	1,930	2,706
2	1,657	1,863	2,118	2,879
3	1,871	2,084	2,339	3,096
4	2,111	2,316	2,570	3,327
5	2,347	2,553	2,808	3,565
6	2,587	2,793	3,048	3,806
7	2,829	3,035	3,291	4,049
8	3,072	3,278	3,534	4,293
9	3,316	3,523	3,779	4,539
10	3,562	3,769	4,025	4,785

TABLE 2. Values of the Dimensionless Thermal Creep Flux Q_T for Various Values of the Rarefaction Parameter δ_T and the Nonisothermal Accommodation Coefficient for Tangential Momentum ε_T

δ_T	ε_T			
	1	0,9	0,8	0,6
0,01	0,7178	0,8640	1,048	1,597
0,02	0,6959	0,8301	0,9970	1,462
0,04	0,6629	0,7804	0,9226	1,305
0,06	0,6373	0,7427	0,8688	1,201
0,08	0,6160	0,7119	0,8237	1,123
0,1	0,5976	0,6851	0,7858	1.056
0,2	0,5294	0,5915	0,6648	0,8297
0,4	0,4464	0,4836	0,5250	0,6234
0,6	0,3925	0,4170	0,4434	0,5053
0,8	0,3529	0,3697	0,3879	0,4285
1	0,3218	0,3330	0,3458	0,3737
2	0,2271	0,2290	0,2304	0,2345
3	0,1765	0,1755	0,1747	0,1734
4	0,1444	0,1426	0,1408	0,1376
5	0,1220	0,1200	0,1179	0,1139
6	0,1056	0,1035	0,1015	0,0972
7	0,0929	0,0910	0,0889	0,0848
8	0,0830	0,0811	0,0792	0,0752
9	0,0750	0,0731	0,0713	0,0675
10	0,0683	0,0666	0,0648	0.0613

ficult to make the discrepancies in Eq. (10) equal to zero in the viscous flow region at an average capillary section which approximates the function ψ_k by a quadratic parabola, i.e., $\psi_k = A_k + B_k r^2$. The validity of this divergence is confirmed also by the fact that the convergence of the results for the Poiseuille flow, where the velocity in the viscous regime is given by a quadratic parabola, is considerably better than for thermal creep, for which the velocity near the wall varies sharply. Therefore, it should be noted that the rate of convergence of the results in the different approximations of the Bubnov—Galerkin method depends appreciably on correct choice of the system of base functions $\varphi_i(r)$.

It can be seen from Tables 1 and 2 that the Poiseuille flux Q_p increases monotonically with decrease of the accommodation coefficient under any flow condition, while a different relation holds for the thermocreep flux Q_T. For example, while Q_T increases with decrease of ε_T in the free-molecular and near-free-molecular flow regime, it decreases in the viscous regime with slip.

Accurate calculations in the second and third approximation show that for $\delta_T = 2.51$ the thermocreep flux Q_T is independent of the coefficient ε_T. This unique dependence of thermocreep on the accommodation coefficient can be explained, if one considers that under isobaric conditions the flux Q_T results from superposition of two oppositely directed fluxes, one of which, Q_t, is due to the nonuniform temperature distribution, and the other, Q_n, is due to the molecular number density gradient. It can be shown, using a substitution for the logarithmic gradients $d \ln n/dz = +d \ln T/dz$ in Eq. (3) of [1], and replacing Eq. (3) by the two independent equations for the perturbation functions $\varphi_t(r)$ and $\varphi_n(r)$ that Q_t and Q_n increase with decrease of ε_T. However, the relation is such that superposition of these fluxes (of the thermocreep Q_T) depends differently on the surface accommodation properties in the different flow regimes.

A comparison of the results obtained with the data of [9] shows that there is superior agreement, within the limits of accuracy of the calculation, only in the case $\varepsilon_{p,T} = 1$, and that there is a systematic divergence (up to 20-30%) for $\varepsilon_{p,T} < 1$. The reason for this discrepancy is the incorrect transformation of the integrals in obtaining integral equations of the type of Eq. (1), which led to the incorrect results in [9]. In this connection, it should be noted that the dependence of Q_T on ε_T in viscous flow with slip is not so weak, so that it can be neglected, as recommended in [9], in comparing theory with experiment.

From the viewpoint of comparing theory and experiment and extracting the appropriate information from the experimental data, either regarding interaction of gases with a surface, or concerning intermolecular interaction, it is important to have an analytical relationship linking the fluxes Q_p and Q_T to the parameters δ_p and δ_T, and the accommodation coefficients ε_p and ε_T. Analytical relations of this kind can be obtained only in the Knudsen number limit.

Thus, analysis of the results shows that in the free-molecular limit the fluxes Q_p and Q_T do not depend on the order of approximation of the Bubnov–Galerkin method, and are given by the formulas

$$Q_p(\delta_p \to 0) = \frac{2-\varepsilon_p}{\varepsilon_p}\,\frac{8}{3\pi^{1/2}}, \quad Q_T(\delta_T \to 0) = \frac{2-\varepsilon_T}{\varepsilon_T}\,\frac{4}{3\pi^{1/2}}. \tag{17}$$

For viscous flow with slip the asymptotic analysis of the results of solution in the second approximation of the Bubnov–Galerkin method lead to the following formulas for Q_p and Q_T:

$$Q_p(\delta_p \gg 1) = \frac{\delta_p}{4} + \frac{2-\varepsilon_p}{\varepsilon_p}\,\frac{\pi^{1/2}}{2}\left(1 + \frac{4-\pi}{2\pi}\,\varepsilon_p\right) + \left[\frac{(\pi-4)^2}{\pi} - \right.$$
$$\left. - \frac{2\pi^2 - 17\pi + 32}{2\pi}\,\varepsilon_p + \frac{(\pi-4)^2}{4\pi}\,\varepsilon_p^2\right]\frac{1}{\delta_p} + \frac{D_p}{\delta_p^2}, \tag{18}$$

$$Q_T(\delta_T \gg 1) = \left(\frac{1}{2} + \frac{\varepsilon_T}{4}\right)\frac{1}{\delta_T} - \frac{\varepsilon_T}{4\pi^{1/2}}\left[2(\pi-1) - (\pi-4)\,\varepsilon_T\right]\frac{1}{\delta_T^2} -$$
$$- \left[\frac{15\pi - 32}{8\pi}\,\varepsilon_T + \frac{8\pi^2 - 83\pi - 160}{16\pi}\,\varepsilon_T^2 - \frac{(\pi-4)^2}{4\pi}\,\varepsilon_T^3\right]\frac{1}{\delta_T^3} + \frac{D_T}{\delta_T^4}. \tag{19}$$

Values of the coefficients D_p and D_T are given in the Appendix.

Comparison of the results of the computations using the asymptotic formulas, Eqs. (18) and (19), with the results of numerical solutions (Tables 1 and 2) shows that with $\delta_p \geq 2$ for Q_p and with $\delta_T \geq 4$ for Q_T the data coincide to within <1%, and with $\delta_p \geq 4$ and $\delta_T \geq 8$, the data practically coincide completely.

From the asymptotic representation of the results with $\delta_{p,T} \gg 1$ in the third approximation of the method one can obtain formulas for Q_p and Q_T in the form

$$Q_p(\delta_p \gg 1) \approx \frac{\delta_p}{4} + \frac{2-\varepsilon_p}{\varepsilon_p}\,\frac{\pi^{1/2}}{2}\left(1 + \frac{4-\pi}{2\pi}\,\varepsilon_p\right), \tag{20}$$

$$Q_T(\delta_T \gg 1) \approx \left(\frac{1}{2} + \frac{\varepsilon_T}{4}\right)\frac{1}{\delta_T} - \frac{3\varepsilon_T}{4\pi^{1/2}}\left[2(\pi-3) - (4-\pi)\,\varepsilon_T\right]\frac{1}{\delta_T^2}. \tag{21}$$

But calculation of terms of higher order with respect to $\delta_{p,T}$ encounters great mathematical difficulties, due primarily to a sharp increase in the volume of the computations.

Comparison of Eqs. (18), (19) and (20), (21) also confirms that the results of solution by the Bubnov–Galerkin method in the different approximations converge better for Poiseuille flow than for thermocreep flow.

Thus, analysis of the results obtained indicates that the Bubnov–Galerkin method is quite efficient for solving a wide circle of rarefied gasdynamic problems. With correct choice of a system of base functions the second approximation of the method is sufficient to obtain results with accuracy on the order of 1%; for increased accuracy of results one must use the third approximation. In addition, with the method one can obtain analytical formulas for macroscopic quantities.

APPENDIX

The coefficients D_p and D_T in Eqs. (18) and (19) are defined by the formulas

$$D_p = 10\pi^{1/2} - \frac{72}{\pi^{1/2}} + \frac{128}{\pi^{3/2}} + \left\{\frac{3\pi^{1/2}}{8}\right\} + \left(\pi^{3/2} - 23\pi^{1/2} + \frac{245}{2\pi^{1/2}} - \right.$$

$$
\begin{aligned}
&-\frac{192}{\pi^{3/2}}-\left\{\frac{3\pi^{1/2}}{8}\right\}\biggr)\varepsilon_p+\biggl(-\pi^{3/2}+15\pi^{1/2}-\frac{68}{\pi^{1/2}}+\frac{96}{\pi^{3/2}}+\\
&+\left\{\frac{3\pi^{1/2}}{32}\right\}\biggr)\varepsilon_p^2+\left(\frac{\pi^{3/2}}{4}-3\pi^{1/2}+\frac{12}{\pi^{1/2}}-\frac{16}{\pi^{3/2}}\right)\varepsilon_p^3,\\
D_{\text{т}}=&\left(4\pi^{1/2}-\frac{25}{\pi^{1/2}}+\frac{32}{\pi^{3/2}}-\left\{\frac{45}{16\pi^{1/2}}\right\}+\left[\frac{105}{16\pi^{1/2}}\right]\right)\varepsilon_{\text{т}}+\\
&+\left(-\frac{17\pi^{1/2}}{2}+\frac{115}{2\pi^{1/2}}-\frac{96}{\pi^{3/2}}+\left\{\frac{15}{16\pi^{1/2}}\right\}-\left\{\frac{9\pi^{1/2}}{16}\right\}\right)\varepsilon_{\text{т}}^2+\\
&+\left(-\frac{\pi^{3/2}}{2}+\frac{19\pi^{1/2}}{2}-\frac{48}{\pi^{1/2}}+\frac{72}{\pi^{3/2}}-\left\{\frac{3}{4\pi^{1/2}}\right\}+\left\{\frac{9\pi^{1/2}}{32}\right\}\right)\varepsilon_{\text{т}}^3+\\
&+\left(\frac{\pi^{3/2}}{4}-3\pi^{1/2}+\frac{12}{\pi^{1/2}}-\frac{16}{\pi^{3/2}}\right)\varepsilon_{\text{т}}^4.
\end{aligned}
$$

Here

$$
\{A\}=A\left(1+\frac{1-\varepsilon_{p,\text{т}}}{2^2}+\frac{(1-\varepsilon_{p,\text{т}})^2}{3^2}+\ldots\right);
$$

$$
[B]=B\left(1+\frac{1-\varepsilon_{p,\text{т}}}{2^3}+\frac{(1-\varepsilon_{p,\text{т}})^2}{3^3}+\ldots\right).
$$

NOTATION

r, R, the radius vector and the capillary radius; c = $|\mathbf{c}|$, the absolute magnitude of the dimensionless molecular velocity vector in a plane normal to the capillary axis; P, T, gas pressure and temperature; δ_p, δ_T, rarefaction parameters; ε_p, ε_T, isothermal and nonisothermal tangential momentum accommodation coefficients; Q_p, Q_T, dimensionless Poiseuille and thermal creep fluxes; $K_n(r, r', \varepsilon_k)$, kernel of the integral equations; $\hat{K}_n\psi_k$, integral operator; $\varphi_i(r)$, base functions; A_k, B_k, C_k, expansion coefficients for the desired functions; a_{ij}, matrix coefficients; α, s, variables in the polar coordinate system; Q_t, Q_n, fluxes due to temperature gradients and molecule number density under isobaric conditions.

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